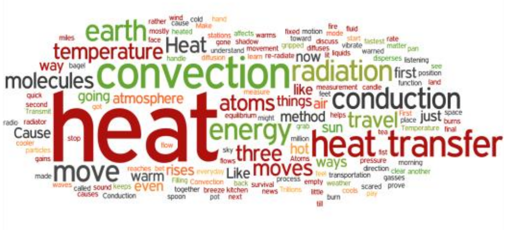
Schule, Abteilung: HTL Mödling, Abteilung Maschineningenieurwesen
Jahrgang: 4. Jahrgang
Lehrer/in: Dipl. Ing. Elisabeth Berger
Schuljahr: 2012/13

Gegenstand: Energie- und Umwelttechnologie
Thema: Heat Transfer

Activity type/s	1. Listen to a video, mark the correct words in a tapescript, sing along the rap song 2. Place and answer questions in written form (language support frame is provided), describe and carry out an experiment 3. Fill in missing terms in a vocabulary list 4. Fill gaps with words from a box 5. Write questions to find missing information in a text 6. Consolidate content by setting up an equation 7. Read a handout, form sentences (writing frame provided) Consolidate content by forming crucial questions and answers 8. Consolidate content by dealing with an assignment of tasks 9. Logical thinking – students watch a video and are asked to find an explanation to a curious example in real life related to conduction in metals
Classroom format	1.- 4. Individual work 5.- 6. Pair work 7.- 8. Teamwork 9. All students together
Time	4 x 50 minutes
Procedure	1. Students listen to a rap song to spark their interest in the fundamentals of heat transfer. Listening a second time they mark the correct words in a transcript. While listening a third time, they are encouraged to join in and sing along! ☺ 2. Students form questions about an experiment and select the correct answers from a list. They write these dialogues in speech bubbles and should be able to describe and carry out the experiment. 3. Students fill in the correct terms from a given list. 4. Students complete sentences to create conceptually correct statements. 5. Students write down questions to fill in the missing words in their text. This exercise is done in pairs (A and B). The only communication between A and B is in written form. 6. Students reconsider the proportionalities in ex. 5 and find out the crucial equation. 7. Students read the core concept of heat conduction and the analogy to electrical conduction, form adequate sentences and extract the crucial information from the theoretical input by forming appropriate questions and answers. 8. Students apply their knowledge about heat conduction to solve tasks related to heat transfer through a composite wall. 9. Students listen to a video to get involved in the topic “Heat Conduction in Metals“. They have to apply their knowledge to answer the question, “Why could your tongue get stuck to metal in the winter?“
Resources	Handout: *Fundamentals of Heat Transfer*
Content-related learning outcome	Students understand the different ways of heat transfer. Students are able to derive and apply Fourier´s Law. Students understand the analogy in heat - and electrical conduction. Students are able to apply their knowledge about heat conduction to solve new tasks. Students attain new perceptions concerning heat conduction in metals and find an answer to a possible problem in real life.
Language-related learning outcome	Students improve their listening and reading abilities. Students can form questions and answers to describe an experiment. Students enrich their vocabulary related to thermodynamics. Students can form questions to obtain missing information. Students develop skills to extract the content of a text and to describe the essentials. Students are encouraged to explain difficult technical content in English and reduce their barrier to express themselves in English.
Source/s	http://www.youtube.com/watch?v=7Y3mfAGVn1c&feature=youtube_gdata_player http://www.youtube.com/watch?v=wit15Iy1xA4 http://www.ccmr.cornell.edu/education/ask/index.html?quid=777



Student's version

Fundamentals of Heat Transfer

Exercise 1a. Listen to the **rap song**

http://www.youtube.com/watch?v=7Y3mfAGVn1c&feature=youtube_gdata_player

Exercise 1b. While listening to the video a second time, mark the correct words in your transcript.

Chorus
There are three ways that heat can travel
But it can *ravel/travel* in any direction
From the sun it moves to the earth, quick as a shadow
And disperses in radiation, conduction, and convection

Verse I
First things first, let's discuss *heat/eat*
What it is before you pay for it to warm your feet
Heat is the *energy/anergy* that flows when atoms and molecules move
It diffuses out toward coolness

Temperature is the measurement of
The average *emotion/motion* of these atoms
And molecules in the atmosphere
So there have it

Now, the first way of transportation of heat energy is *radiation/tradition*
Like sound waves from your radio stations
Filling the empty space around you
Your radiator warms you, but don't start a fire now, I warned you
Heat is going through radiation from the sun
Before it burns you from 93 million miles away
To re-radiate this heat is what the heart/earth does for you
This is survival what this heat is going through
Before it rises to the sky and it's gone so…

Chorus …

Verse II
Conduction is the second method on my list
You can feel it like a hot pot on your fist/first
With the handle gripped I bet you'd rather grab a candle lit
'Cause it's heat through conduction that causes this
Atoms in matter in a fixed position vibrate when heated
Like a pan in the kitchen
Transmit the heat energy to atoms next to them
That are cooler than they are to *case/cause* equilibrium
To even out the temperature the heat has to travel
Up the spoon in your tea in the morning with your bagel

It might burn your hand, but just learn the land of conduction heat energy until you understand
The earth *gains/rains* little from conduction
It's mostly radiation that helps us function and keeps us warm
It's just another way for heat to move
But now let's move on to method three to prove…

Chorus …

Verse III
Convection/connection is the movement of heat in liquids and gasses
To even out the temperature fastest
This process is called diffusion
It's a flow of *articles/particles* cold and hot in a fluid
If you're not listening, then stop what you're doing
'Cause this final mode that heat can move in
And this affects you everyday
It's the *breeze/freeze* at your back and the wind at your face

When heat reaches the earth it rises at a rate
With the air in the place and it moves up till it cools
Trillions of molecules together
Make up the weather that we see on the news
It's how they measure the *treasure/pressure* of the air and the temperature
But convection is how it got there in the atmosphere
So don't be scared. Heat moves in three ways
So we made things clear, it's like…

Exercise 1c. While listening to the video a third time, just join in and sing along! ☺

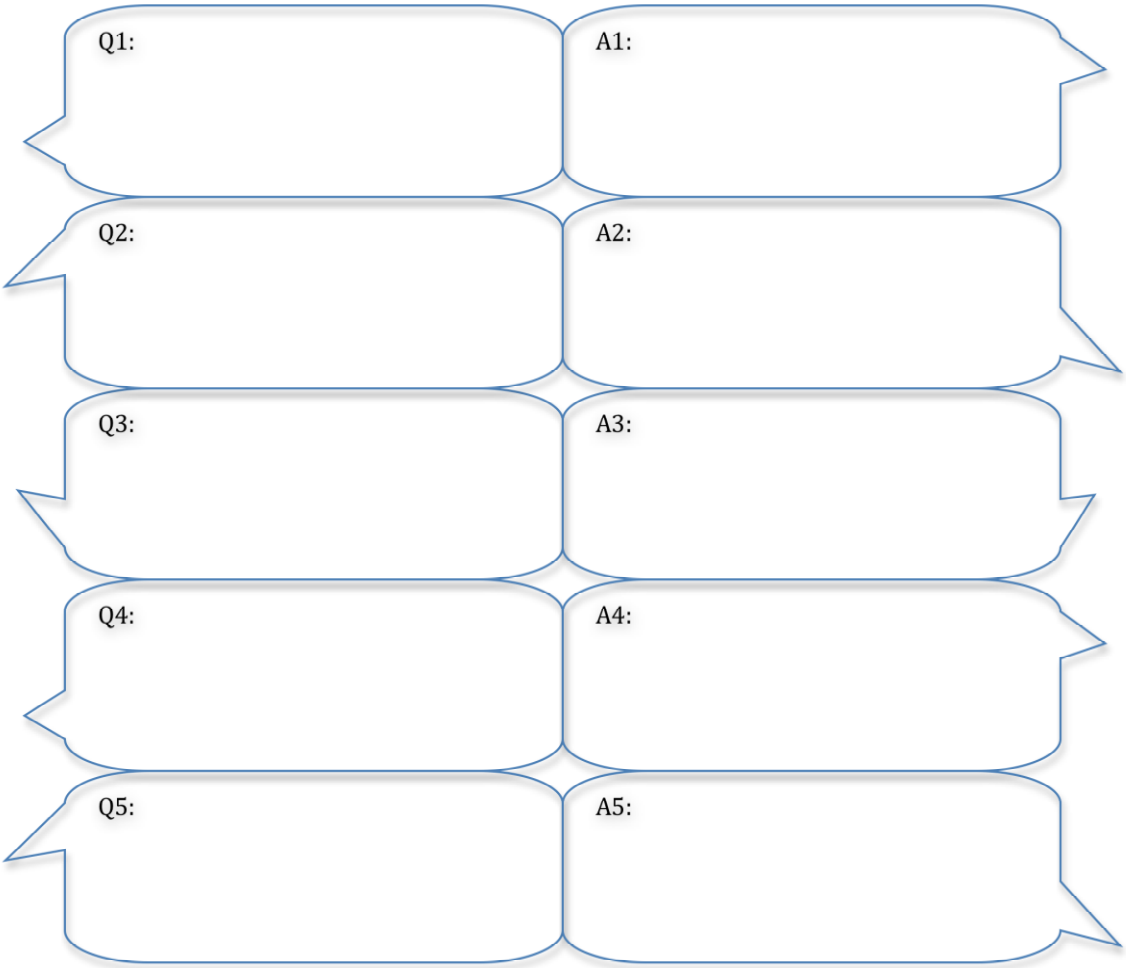
Exercise 2a.
Let´s start! Ask some questions about an experiment:

1	What do we ...	I)	... spoons do we need?
2	How much ...	II)	... do with the hot water?
3	How many ...	III)	... the temperature of the end of the plastic spoon the same as the one of the steel spoon?
4	Will ...	IV)	... water do we need?
5	Is ...	V)	... the end of the steel spoon get warm very fast?

Exercise 2b. Answers (A-E) to the questions (1-5):

A: Just half a litre is enough.
B: No. The steel spoon becomes hot while the end of the plastic spoon doesn´t change the temperature.
C: Yes, it will. Steel is a good thermal conductor.
D: Put some hot water in the glass jar...
E: Two of different materials – one of plastics and one made of steel - to put them partly into the water.

Exercise 2c. Now write these dialogues in the bubbles:



Exercise 2d. Now tell your teacher how to do the experiment.

Exercise 3. Terminology of Heat Transfer
Fill in the correct terms from the list provided.

decreasing	thermal radiation	absolute zero	to maintain	(heat) conduction	insulation
to be linked to sth.	convection	temperature gradient	conductor	heat transfer	

Term	Translation	Comments
	Absoluter Nullpunkt	Absolute zero is defined as 0 K (zero Kelvin) on the Kelvin scale and as −273.15 °C on the Celsius scale, it is the lowest possible temperature.
	Wärmeübertragung	Heat transfer is energy in transit as the result of a temperature difference.
	Wärmeleitung	Conduction is the transfer of heat within a substance by molecular motion.
	Konvektion	Convection is heat transfer that occurs between a surface and a moving fluid, when there is a temperature difference.
	Wärmestrahlung	All surfaces with a temperature above absolute zero emit energy in the form of electromagnetic waves.
	Temperaturgradient	A temperature gradient describes the change in temperature in a given direction. The SI unit is Kelvin per meter (K/m).
	Wärmedämmung	Thermal insulation is the reduction of heat transfer between objects in thermal contact. (What is isolation?)
	(Wärme-, elektr.) Leiter	A conductor is a substance that conducts heat or electricity well.
	halten	The surfaces are maintained at different temperatures.
	verbunden sein mit etw.	The energy is linked to the random motions of the molecules.
	fallend, abnehmend	Energy transfer occurs always in the direction of decreasing temperature.

Exercise 4. Thermal Conduction
Below are some statements, which are **conceptually** correct but **incomplete**. Fill the gaps with the words from the box!

higher	more	fast	less	higher	faster	better

Thermal conduction is the transfer of energy from the ________________________ energetic to the ____________ energetic particles of a substance.

__________________temperatures are associated with _______________________ molecular energy.

Steel is a __________thermal conductor than plastics.

The steel spoon conducts the heat _______________________ than the plastic spoon.

The end of the plastic spoon doesn´t change its temperature significantly.
Plastics doesn´t conduct heat very _______________________, plastics is a thermal insulator.

Exercise 5. "Jail-game": Ask your partner questions to get the missing information.
Get in pairs. One of you reads text A, the other one reads text B. You need to find the missing information to fill in the gaps in your text. However pretend that you and your friend are in jail, but on different sides of a door. You can only get the needed information by exchanging notes under the door. Write your questions on a piece of paper and see what you can find out!

A

______________________ is the transfer of energy from the hot to the cold end of an object. **How fast this happens** is quantified by the rate of heat flow. Heat is denoted Q, the rate of heat flow is Q/t = Q´.

Now, imagine a hot place (jar with hot water) and a cold place (your finger) connected by an object (steel spoon). The object (steel spoon) has a certain length L and a certain cross-sectional area A.

The difference in temperature between the hot and the cold end is denoted ΔT. The higher the difference in temperature, the higher ______________________________. Since this relationship is a direct proportionality, we can write Q´ ~ ΔT.

Conduction also depends on the area of **contact between the hot and cold regions**. The greater the area of contact (the cross-sectional area) , the higher the rate of heat flow, we can write Q´ ~ A.

The shorter the distance between the hot and the cold end is, the higher will be the temperature gradient and the more heat will flow. The rate of heat flow is inversely proportional to ___________________________ ______________________________. This relationship is an inverse proportionality, thus we can write Q´ ~ 1/L.

The steel spoon conducts the heat faster than the plastics spoon with the same dimensions. This difference in the ability to conduct heat is quantified by a factor called **thermal conductivity**, denoted λ. The higher the thermal conductivity, the higher is the rate of heat flow, we can write Q´ ~ λ.

B

Heat conduction is the transfer of energy from the hot to the cold end of an object. _______________ _______________ is quantified by the rate of heat flow. Heat is denoted Q, the rate of heat flow is Q/t = Q´.

Now, imagine a hot place (jar with hot water) and a cold place (your finger) connected by an object (steel spoon). The object (steel spoon) has a certain length L and a certain cross-sectional area A.

The difference in temperature between the hot and the cold end is denoted ΔT. The higher the difference in temperature, the higher **the rate of heat flow**. Since this relationship is a direct proportionality, we can write Q´ ~ ΔT.

Conduction also depends on the area of ____. The greater the area of contact (the cross-sectional area) , the higher the rate of heat flow, we can write Q´ ~ A.

The shorter the distance between the hot and the cold end is, the higher will be the temperature gradient and the more heat will flow. The rate of heat flow is inversely proportional to **the distance (L) between the hot and the cold end**. This relationship is an inverse proportionality, thus we can write Q´ ~ 1/L.

The steel spoon conducts the heat faster than the plastics spoon with the same dimensions. This difference in the ability to conduct heat is quantified by a factor called is quantified by a factor called ______________ ______________________, denoted λ. The higher the thermal conductivity, the higher is the rate of heat flow, we can write Q´ ~ λ.

Exercise 6. Work in pairs
Summarizing all the proportionalities in exercise 5, we can form a single equation. Find out this equation by yourselves!

Exercise 7a. Input for TEAMWORK
Read the information below carefully – then do the tasks given in exercise 7b and exercise 7c!

<u>Fourier´s Law of heat conduction</u>

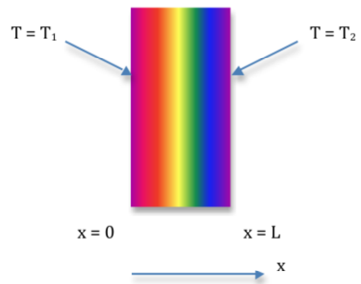
$Q/t = Q' = - \lambda A \, dT/dx$

Heat is conducted in the direction of decreasing temperature.
The constant of proportionality λ is called the **thermal conductivity**.
The thermal conductivity is a function of temperature - the values for familiar materials shown in the table below are for room temperature.

Metals	Ag	Cu	Al	Fe	Steel
λ [W /m· K]	420	390	200	70	50

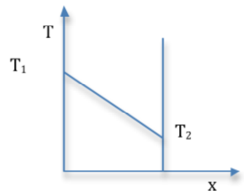
Non-metals	H2O	Air	Brick	Wood	Mineral Wool
λ [W /m· K]	0.6	0.026	0.5 – 1.4	0.2	0.04

<u>Heat Transfer through a Plane Slab (plate)</u>
Let us consider the case of a plane slab, where the ends are kept at constant temperatures T_1 and T_2. The thermal conductivity is assumed to be constant (λ = const.).



Fourier´s Law $Q' = - \lambda A \, dT/dx$
can be rewritten as $dT = - Q' / (\lambda A) \, dx$.
Due to the law of conservation of energy,
Q' needs to be constant.
Integrating this equation gives $T(x) - T_1 = - Q' / (\lambda A) \, x$

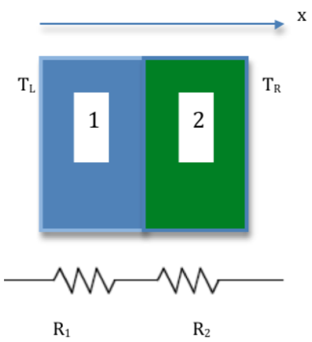
The temperature distribution is $T(x) = T_1 - Q' / (\lambda A) \, x$



This linear variation in temperature is shown for $T_1 > T_2$.
With $\Delta T = T_1 - T_2$, the rate of heat flow can be written in the form **$Q' = \lambda A \Delta T / L$**

<u>Thermal Resistance</u>
There is an electrical analogy with conduction that can be used in problem solving.

Thermal Phenomena	Electrical Phenomena
Fourier´s Law	**Ohm´s Law**
$Q' = \Delta T \lambda A/L = \Delta T / R$	**$I = U / R$**
Rate of Heat Flow Q´	Current I
Temperature Difference ΔT	Voltage Difference U
Thermal Resistance R	**Electrical Resistance R**
$R = L / \lambda A$	**$R = L / \sigma A$**
Length or thickness L	Length L
Cross-sectional area A	Cross-sectional area A
Thermal Conductivity λ	Electrical Conductivity σ
Resistances in Series	**Resistances in Series**
$R = R_1 + R_2 + ...$	**$R = R_1 + R_2 + ...$**



The concept of electrical resistances in series within an electrical circuit allows the analysis of problems such conduction of heat in a composite slab. In the composite slab shown in the figure at the right, the resistances are in series and sum up to **$R = R_1 + R_2$.**
If **T_L** is the temperature at the left, and **T_R** is the temperature at the right,

the rate of heat flow is given by **$Q' = (T_L - T_R) / R$.**

Exercise 7b. Writing Frame

Form as many conceptually correct statements as possible, use the given parts of the sentences. Write them down:

A lower temperature difference between the two sides of a wall	leads to	a higher heat flow.
A larger cross-sectional area of the slab	is linked to	a lower temperature gradient.
A smaller thickness of the plates of a plate heat exchanger	causes	a higher temperature gradient in the insulation layer.
Higher thermal conductivity	is connected with	a lower heat flow.
A higher value of the thermal resistance of the insulating layer of a wall	is coupled to	a lower thermal resistance.

Exercise 7c.

Describe with your own words the analogies in the table of exercise 7a!

To get heat flow in the positive x-direction, the temperature gradient needs to have which sign? Why?

With the help of Fourier's law only, find the dimensional units of the thermal conductivity.

By applying different formats of the Fourier law we can solve different tasks. Please find adequate questions to possible problems and give the respective format of the Fourier law. (e.g. What is the temperature difference, for ...)

Exercise 8.
Heat transfer through a composite wall

The exterior wall of a building consists of 3 layers:
interior plaster: s_{IP} = 1.5 cm, λ_{IP} = 0.8 W/mK
brick: s_B = 24 cm, λ_B = 0.5 W/mK
exterior plaster: s_{EP} = 2.5 cm, λ_{EP} = 0.9 W/mK

The wall surface temperature inside the building is t_{in} = +20°C and outside the building t_{out} = -10°C.
The total area of the building´s exterior wall is 200 m^2.

a) Find the rate of heat loss of the building to the environment!
b) Sketch the resulting temperature distribution in the three layers!

In order to reduce the heat loss of the building to the environment the exterior wall should be provided with an additional insulating layer of mineral wood: s_{MW} = 8 cm, λ_{MW} = 0.04 W/mK

c) How large are the rates of heat loss, if the insulating layer is placed
between the **exterior** plaster and the brick (option A) or
between the **interior** plaster and the brick (option B)?

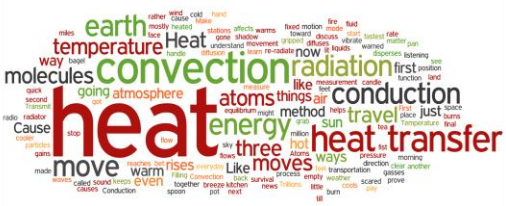
d) Evaluate the temperature distribution within the 4 layers! Sketch a diagram for both option A and B!

e) Which option (A or B) would you prefer? Why?

Exercise 9.
Watch the video (http://www.youtube.com/watch?v=wit15Iy1xA4) and find an answer to the question:

Why does your tongue get stuck to metal in the winter?

Teacher's version



Fundamentals of Heat Transfer

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Exercise 1b. While listening to the video a second time, mark the correct words in your transcript.

Chorus
There are three ways that heat can travel
But it can ~~ravel~~/*travel* in any direction
From the sun it moves to the earth, quick as a shadow
And disperses in radiation, conduction, and convection

Verse I
First things first, let's discuss *heat/~~eat~~*
What it is before you pay for it to warm your feet
Heat is the *energy/~~anergy~~* that flows when atoms and molecules move
It diffuses out toward coolness

Temperature is the measurement of
The average *~~emotion~~/motion* of these atoms
And molecules in the atmosphere
So there have it

Now, the first way of transportation of heat energy is *radiation/~~tradition~~*
Like sound waves from your radio stations
Filling the empty space around you
Your radiator warms you, but don't start a fire now, I warned you
Heat is going through radiation from the sun
Before it burns you from 93 million miles away
To re-radiate this heat is what the ~~heart~~/earth does for you
This is survival what this heat is going through
Before it rises to the sky and it's gone so…

Chorus …

Verse II
Conduction is the second method on my list
You can feel it like a hot pot on your fist/~~first~~
With the handle gripped I bet you'd rather grab a candle lit
'Cause it's heat through conduction that causes this
Atoms in matter in a fixed position vibrate when heated
Like a pan in the kitchen
Transmit the heat energy to atoms next to them
That are cooler than they are to *~~case~~/cause* equilibrium
To even out the temperature the heat has to travel
Up the spoon in your tea in the morning with your bagel

It might burn your hand, but just learn the land of conduction heat energy until you understand
The earth *gains/~~rains~~* little from conduction
It's mostly radiation that helps us function and keeps us warm
It's just another way for heat to move
But now let's move on to method three to prove…

Chorus …

Verse III
Convection/~~connection~~ is the movement of heat in liquids and gasses
To even out the temperature fastest
This process is called diffusion
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And this affects you everyday
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When heat reaches the earth it rises at a rate
With the air in the place and it moves up till it cools
Trillions of molecules together
Make up the weather that we see on the news
It's how they measure the *~~treasure~~/pressure* of the air and the temperature
But convection is how it got there in the atmosphere
So don't be scared. Heat moves in three ways
So we made things clear, it's like…

Exercise 1c. While listening to the video a third time, just join in and sing along! ☺
Check if the students understand all the words and discuss some of the other choices of words in the text.

Exercise 2a.
Let´s start! Ask some questions about an experiment:

1	What do we ...	I)	... spoons do we need?
2	How much ...	II)	... do with the hot water?
3	How many ...	III)	... the temperature of the end of the plastic spoon the same as the one of the steel spoon?
4	Will ...	IV)	... water do we need?
5	Is ...	V)	... the end of the steel spoon get warm very fast?

Exercise 2b. Answers (A-E) to the questions (1-5):

A: Just half a litre is enough.
B: No. The steel spoon becomes hot while the end of the plastic spoon doesn´t change the temperature.
C: Yes, it will. Steel is a good thermal conductor.
D: Put some hot water in the glass jar...
E: Two of different materials – one of plastics and one made of steel - to put them partly into the water.

Exercise 2c. Now write these dialogues in the bubbles:

Q1: What do we do with the hot water?

A1: Put some hot water in the glass jar...

Q2: How much water do we need?

A2: Just half a litre is enough.

Q3: How many spoons do we need?

A3: Two of different materials –
one of plastics and one made of steel -
to put them partly into the water.

Q4: Will the end of the steel spoon
get warm very fast?

A4: Yes, it will.
Steel is a good thermal conductor.

Q5: Is the temperature of the end of the
plastic spoon the same as the one of the
steel spoon?

A5: No. The steel spoon becomes hot
while the end of the plastic spoon
doesn´t change the temperature.

Exercise 2d. Now tell your teacher how to do the experiment.

Exercise 3. Terminology of Heat Transfer
Fill in the correct terms from the list provided.

decreasing	thermal radiation	absolute zero	to maintain	(heat) conduction	insulation
to be linked to sth.	convection	temperature gradient	conductor	heat transfer	

Term	Translation	Comments
absolute zero	Absoluter Nullpunkt	Absolute zero is defined as 0 K (zero Kelvin) on the Kelvin scale and as −273.15 °C on the Celsius scale, it is the lowest possible temperature.
heat transfer	Wärmeübertragung	Heat transfer is energy in transit as the result of a temperature difference.
(heat) conduction	Wärmeleitung	Conduction is the transfer of heat within a substance by molecular motion.
convection	Konvektion	Convection is heat transfer that occurs between a surface and a moving fluid, when there is a temperature difference.
thermal radiation	Wärmestrahlung	All surfaces with a temperature above absolute zero emit energy in the form of electromagnetic waves.
a temperature gradient	Temperaturgradient	A temperature gradient describes the change in temperature in a given direction. The SI unit is Kelvin per meter (K/m).
insulation	Wärmedämmung	Thermal insulation is the reduction of heat transfer between objects in thermal contact. (What is isolation?)
conductor	(Wärme- ; elektr.) Leiter	A conductor is a substance that conducts heat or electricity well.
to maintain	halten	The surfaces are maintained at different temperatures.
to be linked to sth.	verbunden sein mit etw.	The energy is linked to the random motions of the molecules.
decreasing	fallend, abnehmend	Energy transfer occurs always in the direction of decreasing temperature.

Exercise 4. Thermal Conduction
Below are some statements, which are **conceptually** correct but **incomplete**. Fill the gaps with the words from the box!

higher	more	fast	less	higher	faster	better

Thermal conduction is the transfer of energy from the more energetic to the less energetic particles of a substance.

Higher temperatures are associated with higher molecular energy.

Steel is a better thermal conductor than plastics.

The steel spoon conducts the heat faster than the plastic spoon.

The end of the plastic spoon doesn´t change its temperature significantly. Plastics doesn´t conduct heat very fast, plastics is a thermal insulator.

Exercise 5. "Jail-game": Ask your partner questions to get the missing information.
Get in pairs. One of you reads text A, the other one reads text B. You need to find the missing information to fill in the gaps in your text. However pretend that you and your friend are in jail, but on different sides of a door. You can only get the needed information by exchanging notes under the door. Write your questions on a piece of paper and see what you can find out!

A / B

Heat conduction is the transfer of energy from the hot to the cold end of an object. **How fast this happens** is quantified by the rate of heat flow. Heat is denoted Q, the rate of heat flow is Q/t = Q´.

Now, imagine a hot place (jar with hot water) and a cold place (your finger) connected by an object (steel spoon). The object (steel spoon) has a certain length L and a certain cross-sectional area A.

The difference in temperature between the hot and the cold end is denoted ΔT. The higher the difference in temperature, the higher **the rate of heat flow**. Since this relationship is a direct proportionality, we can write $Q' \sim \Delta T$.

Conduction also depends on the area of **contact between the hot and cold regions**. The greater the area of contact (the cross-sectional area) , the higher the rate of heat flow, we can write $Q' \sim A$.

The shorter the distance between the hot and the cold end is, the higher will be the temperature gradient and the more heat will flow. The rate of heat flow is inversely proportional to **the distance (L) between the hot and the cold end**. This relationship is an inverse proportionality, thus we can write $Q' \sim 1/L$.

The steel spoon conducts the heat faster than the plastics spoon with the same dimensions. This difference in the ability to conduct heat is quantified by a factor called **thermal conductivity**, denoted λ. The higher the thermal conductivity, the higher is the rate of heat flow, we can write $Q' \sim \lambda$.

Exercise 6. Work in pairs
Summarizing all the proportionalities in exercise 5, we can form a single equation. Find out this equation by yourselves!

$$Q' \sim \lambda A \Delta T / L$$

Exercise 7a. Input for TEAMWORK
Read the information below carefully – then do the tasks given in exercise 7b and exercise 7c!

Fourier´s Law of heat conduction

$Q/t = Q' = - \lambda A\, dT/dx$

Heat is conducted in the direction of decreasing temperature.
The constant of proportionality λ is called the **thermal conductivity**.
The thermal conductivity is a function of temperature - the values for familiar materials shown in the table below are for room temperature.

Metals	Ag	Cu	Al	Fe	Steel
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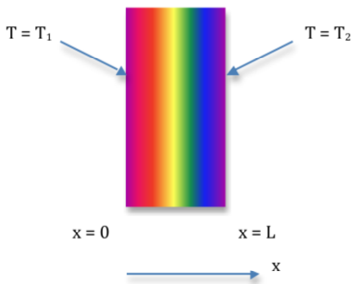
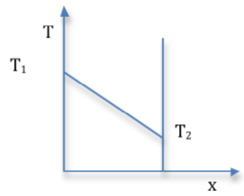
Heat Transfer through a Plane Slab (plate)
Let us consider the case of a plane slab, where the ends are kept at constant temperatures T_1 and T_2. The thermal conductivity is assumed to be constant (λ = const.).



Fourier´s Law $Q' = - \lambda A\, dT/dx$
can be rewritten as $dT = - Q' / (\lambda A)\, dx$.
Due to the law of conservation of energy,
Q' needs to be constant.
Integrating this equation gives $T(x) – T_1 = - Q' / (\lambda A)\, x$

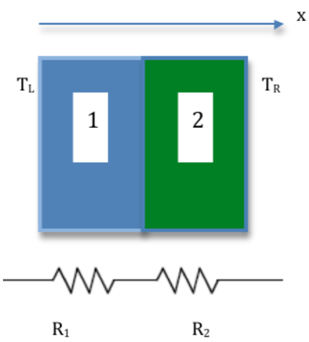
The temperature distribution is $T(x) = T_1 - Q' / (\lambda A)\, x$



This linear variation in temperature is shown for $T_1 > T_2$.
With $\Delta T = T_1 - T_2$, the rate of heat flow can be written in the form **$Q' = \lambda A \Delta T/L$**

Thermal Resistance
There is an electrical analogy with conduction that can be used in problem solving.

Thermal Phenomena	Electrical Phenomena
Fourier´s Law	**Ohm´s Law**
$Q' = \Delta T \lambda A/L = \Delta T / R$	**$I = U / R$**
Rate of Heat Flow Q´	Current I
Temperature Difference ΔT	Voltage Difference U
Thermal Resistance R	**Electrical Resistance R**
$R = L / \lambda A$	**$R = L / \sigma A$**
Length or thickness L	Length L
Cross-sectional area A	Cross-sectional area A
Thermal Conductivity λ	Electrical Conductivity σ
Resistances in Series	**Resistances in Series**
$R = R_1 + R_2 + ...$	**$R = R_1 + R_2 + ...$**



The concept of electrical resistances in series within an electrical circuit allows the analysis of problems such conduction of heat in a composite slab. In the composite slab shown in the figure at the right, the resistances are in series and sum up to **$R = R_1 + R_2$.**
If **T_L** is the temperature at the left, and **T_R** is the temperature at the right,

the rate of heat flow is given by **$Q' = (T_L - T_R) / R$.**

Exercise 7b. Writing Frame

Form as many conceptually correct statements as possible, use the given parts of the sentences. Write them down:

A lower temperature difference between the two sides of a wall	leads to	a lower heat flow. a lower temperature gradient.
A larger cross-sectional area of the slab	is linked to	a lower thermal resistance a higher heat flow.
A smaller thickness of the plates of a plate heat exchanger	causes	a higher heat flow. a lower thermal resistance.
Higher thermal conductivity	is connected with	a lower thermal resistance. a higher heat flow. a lower temperature gradient.
A higher value of the thermal resistance of the insulating layer of a wall	is coupled to	a higher temperature gradient in the insulation layer. a lower heat flow.

Exercise 7c.

Describe with your own words the analogies in the table of exercise 7a!

To get heat flow in the positive x-direction, the temperature gradient needs to have which sign? Why?

With the help of from Fourier's law only, find the dimensional units of the thermal conductivity.

By applying different formats of the Fourier law we can solve different tasks. Please find adequate questions to possible problems and give the respective format of the Fourier law. (e.g. What is the temperature difference, for ...)

Exercise 8.
Heat transfer through a composite wall

The exterior wall of a building consists of 3 layers:
interior plaster: s_{IP} = 1.5 cm, $\lfloor_{IP}$ = 0.8 W/mK
brick: s_B = 24 cm, $\lfloor_B$ = 0.5 W/mK
exterior plaster: s_{EP} = 2.5 cm, $\lfloor_{EP}$ = 0.9 W/mK

The wall surface temperature inside the building is t_{in} = +20°C and outside the building t_{out} = -10°C.
The total area of the building´s exterior wall is 200 m^2.

a) Find the rate of heat loss of the building to the environment!
b) Sketch the resulting temperature distribution in the three layers!

In order to reduce the heat loss of the building to the environment the exterior wall should be provided with an additional insulating layer of mineral wood: s_{MW} = 8 cm, $\lfloor_{MW}$ = 0.04 W/mK

c) How large are the rates of heat loss, if the insulating layer is placed
between the **exterior** plaster and the brick (option A) or
between the **interior** plaster and the brick (option B)?

d) Evaluate the temperature distribution within the 4 layers? Sketch a diagram for both option A and B!

e) Which option (A or B) would you prefer? Why?

Answer:

a) To find the rate of heat loss we apply Fourier´s law for the multilayered wall:

$$\mathbf{Q' = \otimes T / R}$$

$\otimes$**T** is the temperature difference in K between the wall surface temperatures inside and outside the building.
Note that the temperature may be expressed in °C or K as the Fourier Law accounts only for temperature differences which are the same regardless whether °C or K is used.

$$\otimes T = (20 + 273)K - (-10 + 273)K = 30\ K$$

The overall thermal resistance R is $R = R_1 + R_2 + R_3$

The thermal resistance of the first layer is $\mathbf{R_1 = L_1 / \lfloor_1 A_1}$
L_1 is the thickness of the first layer $L_1 = s_{IP}$ = 1.5 cm = 0.015m
$\lfloor_1$ is the thermal conductivity of the first layer $\lfloor_1$ = 0.8 W/(m * K)
$A_1 = A_2 = A_3 = A$ is the area of the wall A= 200m2

We use these values and thereby obtain $R_1 = R_{IP} = 9.4\ 10^{-5}$ K/W

The thermal resistance of the second and the third layer can be computed in an analogous way as

$\mathbf{R_2 = L_2 / \lfloor_2 A_2} = 2.4\ 10^{-3}$ K/W

and

$R_3 = 1.4\ 10^{-4}$ K/W

Adding the values of the individual resistances we get the overall thermal resistance $R = 2.63\ 10^{-3}$ K/W

Therefore the solution for the overall heat rate is
$Q' = \otimes T / R = 11.4\ 10^3$ W = 11.4 kW

b) In order to sketch the temperature distribution we need the temperatures at the contact of the different layers. By the law of conservation of energy no heat can be lost on its way through the composite wall. Therefore the temperature at any place in the wall can be found by applying Fourier´s law in the form of the resistance equation as Q´ is constant throughout the wall.

$Q' = Q'_1 = Q'_2 = Q'_3$

$Q' = \otimes T_1 / R_1 = \otimes T_2 / R_2 = \otimes T_3 / R_3$

The drop in temperature in each layer is therefore

$\otimes T_1 = Q' \oplus R_1$

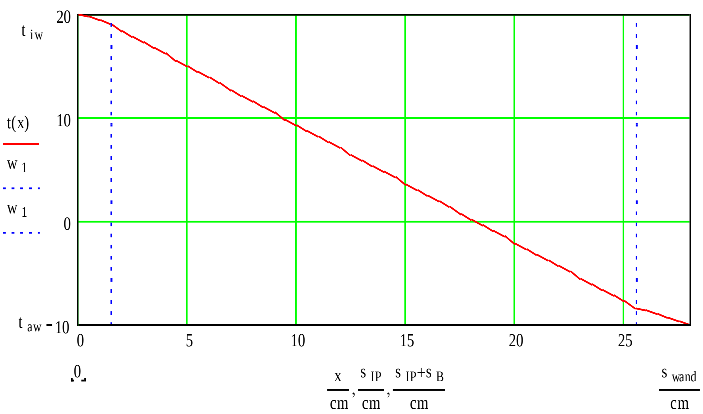
$\otimes T_2 = Q' \oplus R_2$

$\otimes T_3 = Q' \oplus R_3$

This yields **$\otimes T_1$ = 1.07 K or T_2 = 18.93°C**

The same procedure gives **$\otimes T_2$ = 27.35 K or T_3 = -8.42°C** and **$\otimes T_3$ = 1.58 K or T_4 = -10°C**

Thus the temperature distribution through the 3 layered wall is:



c) In order to reduce the heat loss of the building to the environment an additional insulating layer of mineral wood will be applied.
The thermal resistance of the insulating layer is **$R_{IN} = L_{IN} / ⌊_{IN} A_{IN}$**
Using the given values

thickness of the insulating layer $L_{IN} = s_{MW}$ = 8 cm = 0.08m
thermal conductivity $⌊_{IN}$ = 0.04 W/(m * K)
A_{IN} = A = 200m2

we obtain R_{IN} = 0.01 K/W

Adding the values of all the individual resistances we find the overall thermal resistance for both options A and B

$R_A = R_1 + R_2 + \mathbf{R_{IN}} + R_3$ = 0.013 K/W

$R_B = R_1 + \mathbf{R_{IN}} + R_2 + R_3$ = 0.013 K/W

and we easily recognize that $R_A = R_B = R$
The overall thermal resistance doesn´t depend on whether the insulating layer is placed
between the **exterior** plaster and the brick (option A) or
between the **interior** plaster and the brick (option B)!

The result for the overall heat rate is therefore **$Q' = \otimes T / R = 2.375\ 10^3$ W = 2.375 kW**

There is still a heat flow to the outside, but with insulation the loss is only 2.375 kW compared to 11.395 kW without insulation.

d)
The temperature drops in each layer using Q´ as calculated above are now

$\otimes T_1 = Q' \oplus R_1 = 0.22K$

$\otimes T_2 = Q' \oplus R_2 = 5.7\ K$

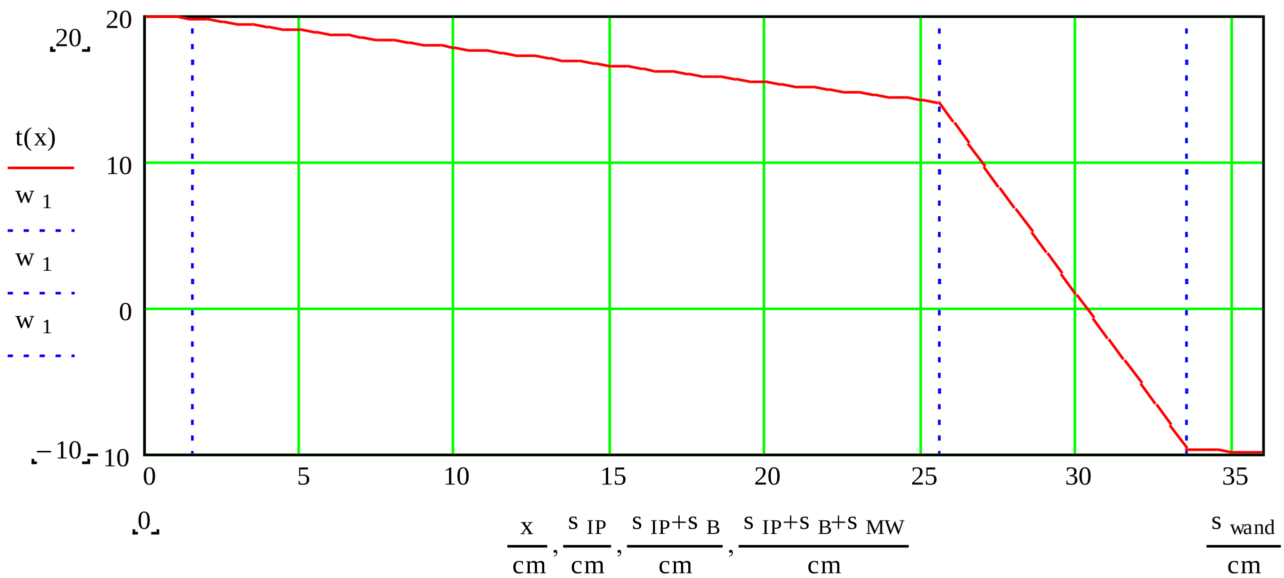
$\otimes T_3 = Q' \oplus R_3 = 0.33\ K$

$\otimes T_{IN} = Q' \oplus R_{IN} = 23.75\ K$

The temperature distribution however depends on the position of the insulating layer:

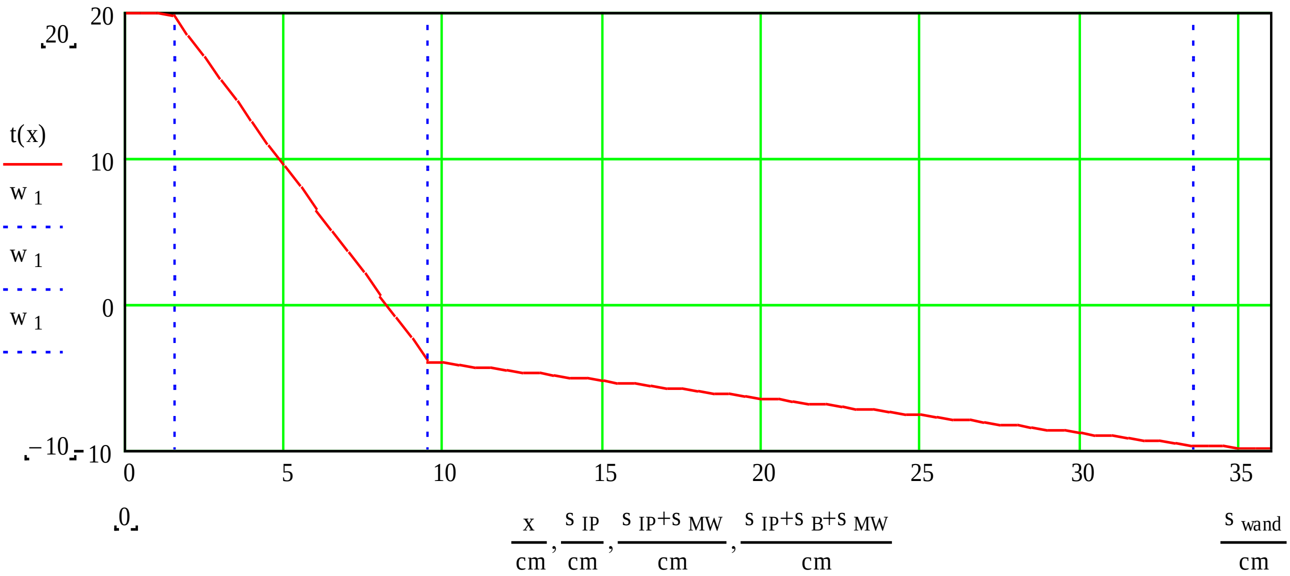
Option A (insulating between the **exterior** plaster and the brick)

$T_2 = T_1 - \otimes T_1 = 19.78$ °C
$T_3 = T_2 - \otimes T_2 = 14.09$ °C
$T_4 = T_3 - \otimes T_{IN} = -9.67$ °C
$T_5 = T_4 - \otimes T_3 = -10$ °C



The same procedure for option B (insulating between the **interior** plaster and the brick) gives

$T_2 = T_1 - \otimes T_1 = 19.78$ °C
$T_3 = T_2 - \otimes T_{IN} = -3.97$ °C
$T_4 = T_3 - \otimes T_2 = -9.67$ °C
$T_5 = T_4 - \otimes T_3 = -10$ °C



As sketched in both diagrams, the largest temperature drop is across the insulating layer even though the brick layer is much thicker!

e) Which option do you prefer? Why?
Insulating according option A results in a high temperature drop in an outside layer of the wall, whereas option B effects the same drop in an inner layer of the wall.

Insulation at the inner side of the wall brings the lower temperatures closer to the inside. This results in the fact that the dew point of the humid air also comes closer to the inside. If the inner insulation is so effective that the contact temperature between insulation and the brick wall lands below the dew point the brick wall will become wet and the risk of formation of mold appears.

Exercise 9.
Watch the video (http://www.youtube.com/watch?v=wit15Iy1xA4) and find an answer to the question:

Why does your tongue get stuck to metal in the winter?

When the temperature of the metal is well below 0 °C, the waterfilm in contact with the metal freezes immediately to ice. From a thermodynamical point of view the system tongue plus waterfilm forms a two-layered slab. As the waterfilm is very thin the contact temperature between waterfilm and tongue easily ends up below 0 °C, which causes the tongue to stick to the formed ice.

For a more extensive explanation please refer to:

http://www.ccmr.cornell.edu/education/ask/index.html?quid=777