Jump discontinuity

1 Consider $f(x)=\dfrac{|x|}{x}$.

a. State the domain of $f(x)$.

b. Is $f(x)$ continuous everywhere? If $f(x)$ is discontinuous at any value, determine whether such value is a removable discontinuity.

c. Graph the function.

a. The domain of $f(x)$ is $(-\infty\,;\,0)\cup(0\,;\,+\infty)$.

b. To determine the type of discontinuity evaluate the limit of $f(x)$ as x approaches zero from the right. As $x\to 0^+$, we can replace $|x|$ by x.

$$\lim_{x\to 0^+}\frac{|x|}{x}=\lim_{x\to 0^+}\frac{x}{x}=1$$

Now evaluate the limit of $f(x)$ as x approaches zero from the left. As $x\to 0^-$, we can replace $|x|$ by $-x$.

$$\lim_{x\to 0^-}\frac{|x|}{x}=\lim_{x\to 0^-}\frac{-x}{x}=-1$$

Thus $\lim\limits_{x\to 0^+}\dfrac{|x|}{x}\neq\lim\limits_{x\to 0^-}\dfrac{|x|}{x}$. Hence $f(x)$ is discontinuous at $x=0$ and this discontinuity is not removable.

c. Write $f(x)$ as

$$f(x)=\begin{cases}\dfrac{x}{x} & \text{if } x>0\\[2mm] \dfrac{-x}{x} & \text{if } x<0\end{cases}\longrightarrow f(x)=\begin{cases}1 & \text{if } x>0\\ -1 & \text{if } x<0\end{cases}$$

Now it is easy to sketch the graph of $f(x)$.

The discontinuity shown in **FIGURE 1** is called *jump discontinuity*.

The jump is $\left|\lim\limits_{x\to 0^+}\dfrac{|x|}{x}-\lim\limits_{x\to 0^-}\dfrac{|x|}{x}\right|=2.$

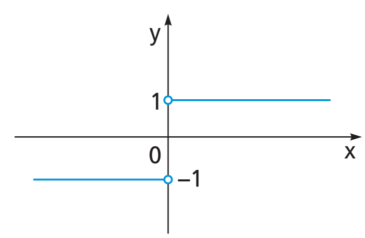

FIGURE 1

What does it mean?

Removable discontinuity Punto di singolarità eliminabile. Singolarità di terza specie

As *x* approaches *c* from the right Per *x* tendente a *c* da destra

As *x* approaches *c* from the left Per *x* tendente a *c* da sinistra

Jump discontinuity L'espressione corrisponde a «singolarità di prima specie»

Jump È il «salto» nei valori della funzione passando da sinistra a destra del punto di singolarità di prima specie