# English for Maths

## Possible graph of a function

**1** Sketch a possible graph for the function $y = f(x) = \dfrac{x^2+1}{1-x^2}$.

**a.** State the domain restrictions and find the vertical asymptotes.
**b.** Determine whether $f(x)$ is an even function or an odd function.
**c.** Find the $x$-intercepts.
**d.** Find the $y$-intercept.
**e.** Determine the sign of $f(x)$.
**f.** Determine the behaviour of the graph as $x$ approaches the points at which $f(x)$ is not defined.
**g.** Determine the end behaviour of the graph.
**h.** Sketch the graph.

**a.** State the domain restrictions

$1 - x^2 \neq 0 \longrightarrow x \neq \pm 1$

The domain is $(-\infty\,;\,-1) \cup (-1\,;\,1) \cup (1\,;\,+\infty)$.

The function is not defined at $x = 1$ and $x = -1$ and therefore the graph has vertical asymptotes at each such value (**FIGURE 1**).

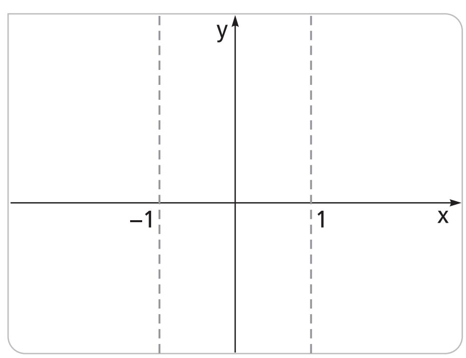


**FIGURE 1**

In **FIGURE 1** the vertical asymptotes are represented by dashed vertical lines. The graph separates into three branches.

**b.** Determine whether $f(x)$ is an even function:

$$f(-x) = \frac{(-x)^2+1}{1-(-x)^2} \longrightarrow f(-x) = \frac{x^2+1}{1-x^2}$$

Since $f(x) = f(-x)$, the original function is even and is symmetric about the $y$-axis.

**c.** Find the $x$-incercepts, so let $y = 0$:

$$\frac{x^2+1}{1-x^2} = 0$$

Solve for $x$:

$x^2 + 1 = 0$

Calculate the discriminant:

$\Delta = -4$

$\Delta < 0$ and therefore there are no real zeroes; so $f(x)$ has no $x$-intercepts.

**d.** Find the $y$-intercept, so let $x = 0$:

$$y = \frac{(0)^2 + 1}{1 - (0)^2} \longrightarrow y = 1$$

The $y$-intercept is $P(0\,;\,1)$.

**e.** Use a sign chart to determine the sign of $f(x)$ (**FIGURE 2**).

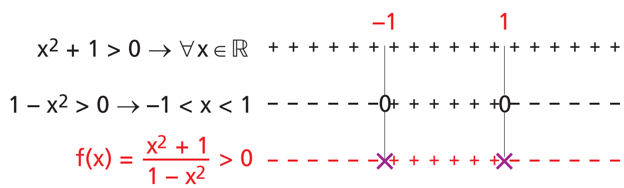


**FIGURE 2**

From the sign chart we see that

$f(x) < 0$ for $x \in (-\infty\,;\,-1)$; that is, $f(x)$ is *below* the $x$-axis for $x \in (-\infty\,;\,-1)$

$f(x) > 0$ for $x \in (-1\,;\,1)$; that is, $f(x)$ is *above* the $x$-axis for $x \in (-1\,;\,1)$

$f(x) < 0$ for $x \in (1\,;\,+\infty)$; that is, $f(x)$ is *below* the $x$-axis for $x \in (1\,;\,+\infty)$ (**FIGURE 3**)

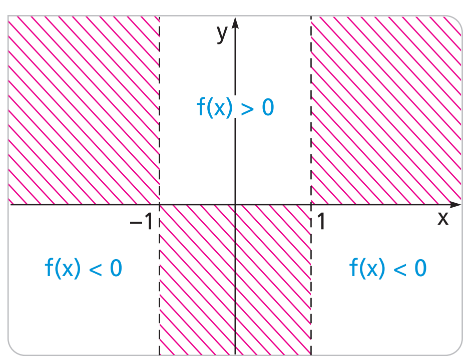


**FIGURE 3**

**f.** Determine the behaviour of $f(x)$ as $x$ approaches $-1$

$$\lim_{x \to -1^-} \frac{x^2 + 1}{1 - x^2} = -\infty$$

$$\lim_{x \to -1^+} \frac{x^2 + 1}{1 - x^2} = +\infty$$

Determine the behaviour of $f(x)$ as $x$ approaches 1

$$\lim_{x \to 1^-} \frac{x^2 + 1}{1 - x^2} = +\infty$$

$$\lim_{x \to 1^+} \frac{x^2 + 1}{1 - x^2} = -\infty$$

In **FIGURE 4**, the graph decreases without bound as $x$ approaches $-1$ from the left; and the graph increases without bound as $x$ approaches $-1$ from the right. The graph increases without bound as $x$ approaches 1 from the left and the graph decreases without bound as $x$ approaches 1 from the right.

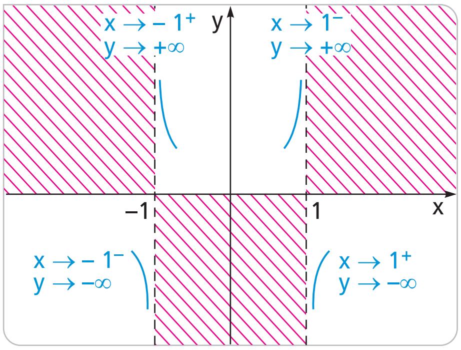


**FIGURE 4**

**g.** Determine the end behaviour of the graph

$$\lim_{x \to \pm\infty} \frac{x^2+1}{1-x^2} = \lim_{x \to \pm\infty} \frac{x^2\left(1+\dfrac{1}{x^2}\right)}{x^2\left(\dfrac{1}{x^2}-1\right)} = -1$$

Thus $y=-1$ is a horizontal asymptote.

In **FIGURE 5**, the graph get closer and closer to the horizontal line $y=-1$ as $x$ increases or decreases without bound.

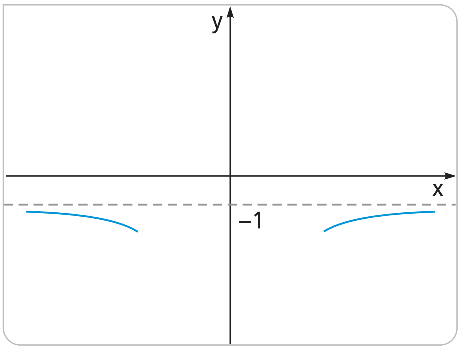


**FIGURE 5**

**h.** Now it is easy to sketch the required graph (**FIGURE 6**).

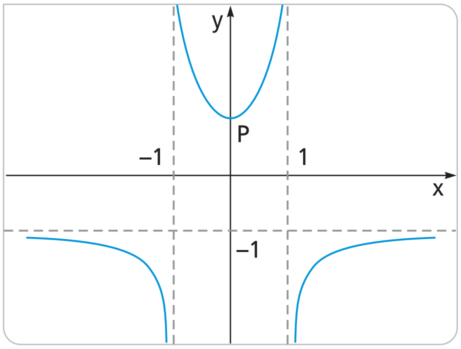


**FIGURE 6**

## What does it mean?

**To sketch a possible graph for $f(x)$** Disegnare il grafico probabile di $f(x)$

**Even function** Funzione pari

**Odd function** Funzione dispari

***x*-intercepts** Intersezioni con l'asse delle $x$

**End behaviour** È il comportamento di una funzione al tendere di $x$ all'infinito

**Symmetric about the *y*-axis** Simmetrico rispetto all'asse delle $y$

**Dashed line** Retta tratteggiata

**Discriminant** Discriminante

**Sign chart** È lo schema usato per lo studio del segno di una funzione. Viene anche detto **sign diagram**

**To increase (or decrease) without bound** Crescere (o descrescere) illimitatamente