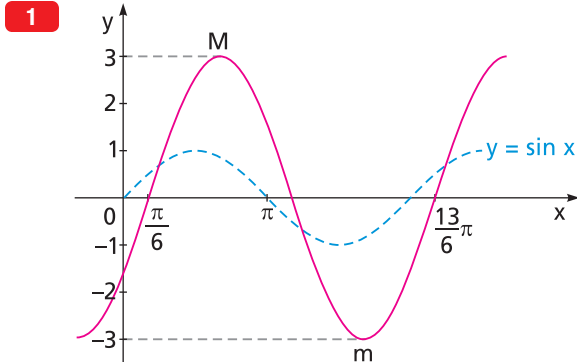


Trigonometric functions

Determine the equation of the given graphs.



Use the basic sine function $y = \sin x$. It can be written in standard form $y = A \sin(\omega x + \varphi) + D$ or in shifted form $y = A \sin\left[\omega\left(x + \frac{\varphi}{\omega}\right)\right] + D$

where:

- $A = \left|\frac{M - m}{2}\right|$ is the amplitude of the graph or maximum ordinate, M is the maximum value of the graph, m is the minimum value of graph;
- $\omega = \frac{2\pi}{T}$ is the angular frequency, T is the period;
- φ is the initial phase or phase angle;
- $\frac{\varphi}{\omega}$ is the magnitude and the direction of the horizontal shift, opposite the given sign;
- $D = \frac{M + m}{2}$ is the vertical shift and the average value of the function.

From the graph we see that:

- $A = \left|\frac{3 + 3}{2}\right| = 3$;
- since $D = \frac{3 - 3}{2} = 0$ there is not vertical shift;
- the basic graph has been shifted to the right $\frac{\pi}{6}$ therefore $\varphi = \frac{\pi}{6}$;
- the period is $T = \frac{13\pi}{6} - \frac{\pi}{6} = 2\pi$;
- the angular frequency is $\omega = \frac{2\pi}{2\pi} = 1$.

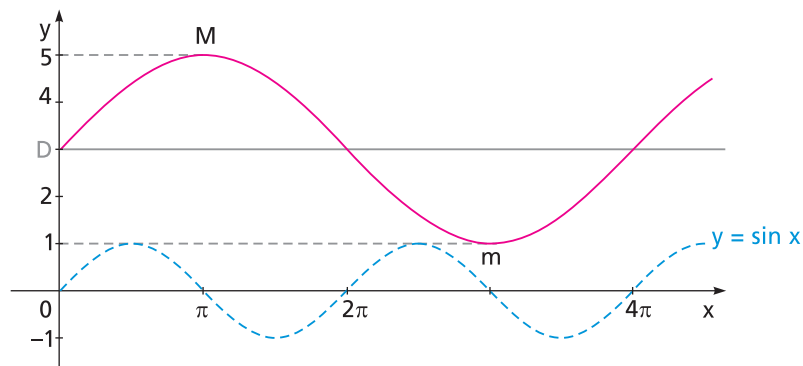
Now substitute $A = 3$, $\omega = 1$, $\frac{\varphi}{\omega} = -\frac{\pi}{6}$ (opposite the given sign), $D = 0$ into the equation $y = A \sin\left[\omega\left(x + \frac{\varphi}{\omega}\right)\right] + D$

$$y = 3 \sin \left[(1) \left(x - \frac{\pi}{6} \right) \right] + 0$$

The equation of the given graph is

$$y = 3 \sin \left(x - \frac{\pi}{6} \right)$$

2



Use the basic function $y = \sin x$.

From the graph we see that:

- $D = \frac{5+1}{2} = 3 \rightarrow$ the average value of the function (and the vertical shift) is $D = 3$;
- $\varphi = 0 \rightarrow$ the graph begins at the average value ($D = 3$), therefore the basic graph has not been horizontally shifted;
- $A = \left| \frac{5-1}{2} \right| = 2$;
- $T = 4\pi$;
- $\omega = \frac{2\pi}{4\pi} = \frac{1}{2} \rightarrow$ the basic graph $y = \sin x$ has been stretched horizontally.

Now substitute $A = 2$, $\omega = \frac{1}{2}$, $\frac{\varphi}{\omega} = 0$, $D = 3$ into $y = A \sin \left[\omega \left(x + \frac{\varphi}{\omega} \right) \right] + D$

$$y = 2 \sin \left[\frac{1}{2} (x + 0) \right] + 3$$

The equation of the graph is

$$y = 2 \sin \frac{1}{2} x + 3$$

What does it mean?

Amplitude Ampiezza

Angular frequency Pulsazione

Initial phase or phase angle Fase iniziale, fase o sfasamento

Horizontal shift Traslazione orizzontale

Vertical shift Traslazione verticale

Graph stretched horizontally Grafico dilatato orizzontalmente